This document describes an example calculation of a reserve factor (RF) or margin of safety (MS) using the "simple" structural model, which has been made available for the AIAA Challenge Problem[1].

The calculation essentially consists of three steps. The first step of the exercise is to find the interface loads at the discrete interface points utilizing the structural model, which can be viewed of as a transfer function that distributes the applied aerodynamic loads to the individual interface points. Application of this transfer function requires the pressure resultant forces and moments to be available at some reference point. Second step in the process is to determine the strength of the pin joint connections which together constitute the attachment to the fuselage. The analysis method that will be followed is given in Chapter 9 of the *Air Force Stress Analysis Manual*[2], which is open to the public and can be downloaded from the internet. Once both the applied loads and the attachment strengths have been determined, the final step of computing the RF or MS can be performed.

In the following each of these three steps will be detailed for an example calculation for one specific interface point, but first the structural model will be discussed with the necessary level of rigor.

Simple Structural Model

A detailed FEM model of the radome assembly is also available for the challenge problem (see Figure 1). This model requires an (off the shelf) FEM code to run and produce the desired interface loads at the nine fuselage attachment points. To enable the calculation of the interface loads without the necessity of utilizing a general-purpose FEM code, a simplified approach can be followed which is based on a classical mechanics approach considering the minimization of the total energy of the loaded radome assembly system, where the calculations merely involve some elementary matrix algebra (which can, for example, be performed in a spreadsheet program).

[1] "Challenge Problem Overview for the Certification by Analysis Uncertainty Quantification Discussion Group," Phillipp Bekemeyer et al., July 2025. (https://doi.org/10.2514/6.2025-3107)
[2] "Stress Analysis Manual," Gene E. Maddux et. al., AFFDL-TR-69-42, August 1969. (https://apps.dtic.mil/sti/pdfs/AD0759199.pdf)

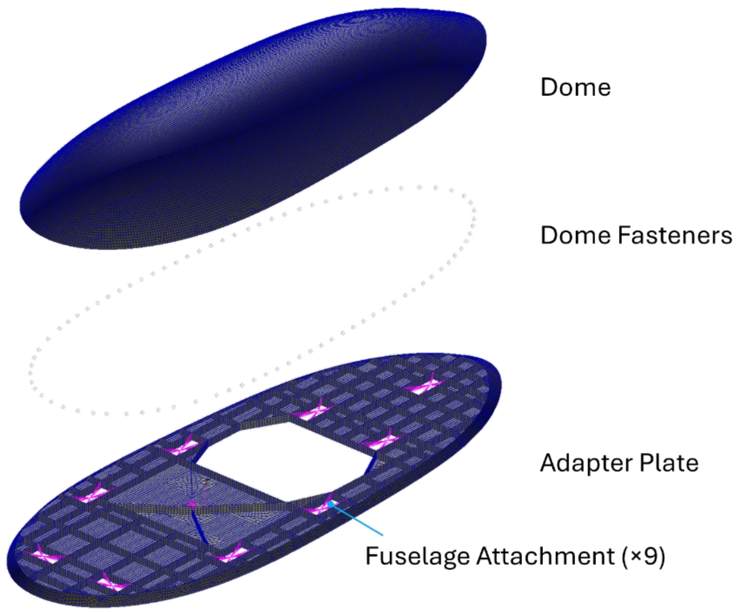
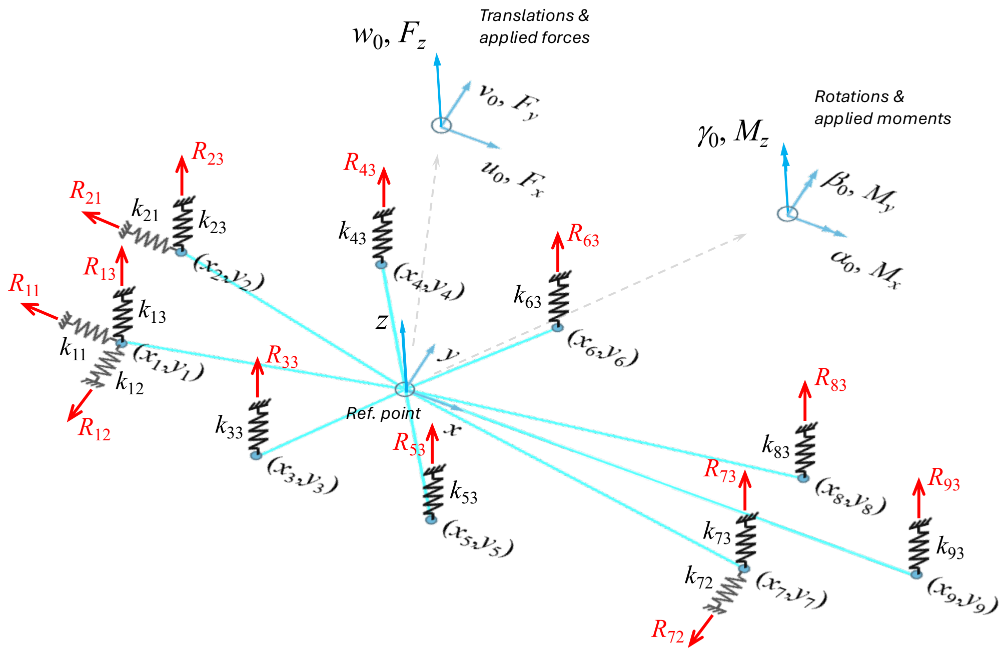


Figure 1 Detailed FEM model of radome assembly showing the major components

The simple model treats the radome assembly as a rigid body that rotates with respect to some (arbitrary) reference point. This body is restrained by linear springs located at the fuselage interface points. These springs collectively represent the stiffness of the radome assembly; the values of the corresponding spring stiffnesses are such that (approximately) the same interface loads are obtained as for the detailed FEM model for the same given loading conditions.



Point	x [in]	y [in]
1	-32.5	-8
2	-32.5	8
3	-11.5	-16.1
4	-11.5	16.1
5	11.5	-16.1
6	11.5	16.1
7	37.5	-8
8	37.5	8
9	57.2	0

Figure 2 Simple structural model of radome assembly featuring springs at the fuselage attachment points

The simple model has a total of six degrees of freedom (DOF) at the reference point; the three translations u_0,v_0, and w_0; and the three rotations α_0, β_0, and γ_0, as illustrated in Figure 2. The displacements at interface point i follow from the translations and rotations at the reference point:

$$u_{i1} = u_0 - y_i\gamma_0$$
$$u_{i2} = v_0 + x_i\gamma_0$$
$$u_{i3} = w_0 + y_i\alpha_0 - x_i\beta_0$$

Equation 1

where the 1, 2, and 3 in the subscripts refer to the x-, y-, and z-direction, respectively (coincident with the direction of the springs). Radome pressures obtained from a CFD run are summed at the reference point to produce the force and moment resultants F_x, F_y, F_z, M_x, M_y, and M_z (see Figure 2), which are then applied to the structural model.

Introducing the displacement and load vectors

$$\{u\} = \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \alpha_0 \\ \beta_0 \\ \gamma_0 \end{Bmatrix} \quad , \quad \{F\} = \begin{Bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$$

Equation 2

the mathematical form of the model becomes

$$\{F\} = [K]\{u\}$$
$$\Rightarrow \{u\} = [K]^{-1}\{F\}$$

Equation 3

where $[K]$ is the stiffness matrix given by

$$[K] = \begin{bmatrix} \sum_{i=1}^{9} k_{i1} & 0 & 0 & 0 & 0 & -\sum_{i=1}^{9} k_{i1} y_i \\ 0 & \sum_{i=1}^{9} k_{i2} & 0 & 0 & 0 & \sum_{i=1}^{9} k_{i2} x_i \\ 0 & 0 & \sum_{i=1}^{9} k_{i3} & \sum_{i=1}^{9} k_{i3} y_i & -\sum_{i=1}^{9} k_{i3} x_i & 0 \\ 0 & 0 & \sum_{i=1}^{9} k_{i3} y_i & \sum_{i=1}^{9} k_{i3} y_i^2 & -\sum_{i=1}^{9} k_{i3} x_i y_i & 0 \\ 0 & 0 & -\sum_{i=1}^{9} k_{i3} x_i & -\sum_{i=1}^{9} k_{i3} x_i y_i & \sum_{i=1}^{9} k_{i3} x_i^2 & 0 \\ -\sum_{i=1}^{9} k_{i1} y_i & \sum_{i=1}^{9} k_{i2} x_i & 0 & 0 & 0 & \sum_{i=1}^{9} \left(k_{i2} x_i^2 + k_{i1} y_i^2 \right) \end{bmatrix}$$

Equation 4

Hence, the elements of the stiffness matrix involve the individual spring stiffnesses at the interface points, k_{ij}, and the coordinates (x_i, y_i).of these points.

According to Equation 3, the stiffness matrix $[K]$ must be inverted, after which the displacement vector is readily calculated from pre-multiplication with the load vector. Once the individual translation and rotation components at the reference point are known, the displacements (existing of translations only representing the elongation of the springs) at the interface points in each of the spring directions can be calculated from Equation 1; then, the spring forces (which are the reaction or interface loads, R_{ij}, in Figure 2) follow from multiplication of the individual spring stiffnesses and the appropriate displacements. Thus, the interface loads are given by

$$R_{ij} = k_{ij} u_{ij} \quad , \quad i = 1, 2, \dots 9, j = 1, 2, 3$$

Equation 5

Note: not each u_{ij} has a spring associated with it due to the motion allowed in that particular direction per design; all interface points have a reaction (spring) in the z-direction.

As an example for the purpose of demonstration, values of the individual spring stiffnesses are given in Table 1 below.

Table 1 Example spring stiffnesses for the simple structural model in lb/in (N/mm)

I/F Point i	k_{i1}	k_{i2}	k_{i3}
1	100000 (17510)	1000000 (175130)	15300 (2690)
2	1588000 (278020)	-	15300 (2690)
3	-	-	10000 (1750)
4	-	-	10000 (1750)
5	-	-	11200 (1970)
6	-	-	11200 (1970)
7	-	1802000 (315660)	0
8	-	-	0
9	-	-	20300 (3550)

The given spring stiffnesses are representative of a rigid grounding (i.e., an infinite stiffness of the fuselage backup structure). To account for a compliant attachment base, spring stiffnesses may be established representing the fuselage backup structure. Denoting these spring stiffnesses at the attachment points by K_{ij}, these can then be combined with the radome assembly stiffnesses k_{ij} to form an effective spring stiffness (springs in series) as follows:

$$k_{ij,\mathrm{eff}} = \left(\frac{1}{k_{ij}} + \frac{1}{K_{ij}}\right)^{-1}$$

Equation 6

From this formula the effective spring stiffnesses are bounded on the lower end by the fuselage backup stiffnesses K_{ij} for low values thereof, and by the radome assembly stiffnesses k_{ij} for a rigid attachment (i.e. $K_{ij} \to \infty$) on the higher end.

To summarize, the steps to be followed for determining the fuselage interface loads using the simple structural model are:

1. Calculate the stiffness matrix from the individual spring stiffnesses and the location of the interface points (Equation 4).
2. Invert the stiffness matrix (Equation 3).
3. Multiply the inverted stiffness matrix and the applied load vector to yield the translations and rotations at the reference point (Equation 3).
4. Calculate the displacements at the interface points. (Equation 1).
5. Calculate the spring forces (interface loads) using matching spring stiffnesses and displacements (Equation 5).

Example calculation

Step 1

Applying the spring stiffnesses given in Table 1 and the coordinates of the nine attachment points (per Figure 2) to Equation 4, the stiffness matrix becomes

$$[K] = \begin{bmatrix} 1.688\cdot 10^{6} & 0 & 0 & 0 & 0 & -1.190\cdot 10^{7} \\ 0 & 2.802\cdot 10^{6} & 0 & 0 & 0 & 3.508\cdot 10^{7} \\ 0 & 0 & 9.330\cdot 10^{4} & -6.003\cdot 10^{0} & -1.935\cdot 10^{5} & 0 \\ 0 & 0 & 0 & 1.295\cdot 10^{7} & 0 & 0 \\ 0 & 0 & -1.935\cdot 10^{5} & 0 & 1.043\cdot 10^{8} & 0 \\ -1.190\cdot 10^{7} & 3.508\cdot 10^{7} & 0 & 0 & 0 & 3.698\cdot 10^{9} \end{bmatrix}$$

giving an inverse of (Step 2)

$$[K]^{-1} = \begin{bmatrix} 6.081\cdot 10^{-7} & -2.780\cdot 10^{-8} & 0 & 0 & 0 & 2.221\cdot 10^{-9} \\ -2.780\cdot 10^{-8} & 4.062\cdot 10^{-7} & 0 & 0 & 0 & -3.942\cdot 10^{-9} \\ 0 & 0 & 1.076\cdot 10^{-5} & 0 & 1.997\cdot 10^{-8} & 0 \\ 0 & 0 & 0 & 7.723\cdot 10^{-8} & 0 & 0 \\ 0 & 0 & 1.997\cdot 10^{-8} & 0 & 9.629\cdot 10^{-9} & 0 \\ 2.221\cdot 10^{-9} & -3.942\cdot 10^{-9} & 0 & 0 & 0 & 3.149\cdot 10^{-10} \end{bmatrix}$$

Consider applied limit loads (which are the resultants from a particular aerodynamic pressure distribution on the dome) of

$$F_z = 4500 \text{ lb}, M_y = -56700 \text{ in-lb, and } F_x = F_y = M_x = M_z = 0$$

the displacement vector at the reference point becomes (Step 3)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \alpha_0 \\ \beta_0 \\ \gamma_0 \end{Bmatrix} = \begin{bmatrix} 6.081\cdot 10^{-7} & -2.780\cdot 10^{-8} & 0 & 0 & 0 & 2.221\cdot 10^{-9} \\ -2.780\cdot 10^{-8} & 4.062\cdot 10^{-7} & 0 & 0 & 0 & -3.942\cdot 10^{-9} \\ 0 & 0 & 1.076\cdot 10^{-5} & 0 & 1.997\cdot 10^{-8} & 0 \\ 0 & 0 & 0 & 7.723\cdot 10^{-8} & 0 & 0 \\ 0 & 0 & 1.997\cdot 10^{-8} & 0 & 9.629\cdot 10^{-9} & 0 \\ 2.221\cdot 10^{-9} & -3.942\cdot 10^{-9} & 0 & 0 & 0 & 3.149\cdot 10^{-10} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 4500 \\ 0 \\ -56700 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.0473 \\ 0 \\ -0.00046 \\ 0 \end{Bmatrix}$$

With the displacements at the reference point known, Step 4 can 5 can be performed by employing Equation 1 and Equation 5 to obtain the translation at each individual spring (representing the elongation or shortening) and the associated spring force, respectively. The calculation of the spring forces at each interface point follows next.

Point 1

Displacements:

$$u_{11} = 0 - (-8) \times 0 = 0$$

$$u_{12} = 0 - (-32.5) \times 0 = 0$$

$$u_{13} = 0.0473 + (-8) \times 0 - (-32.5) \times (-0.00046) = 0.0324 \text{ in}$$

Spring forces (interface loads):

$$R_{11} = 100000 \times 0 = 0$$

$$R_{12} = 1000000 \times 0 = 0$$

$$R_{13} = 15300 \times 0.0324 = 496 \text{ lb (tension; = up)}$$

Point 2

Displacements:

$$u_{21} = 0 - (-8) \times 0 = 0$$

$$u_{23} = 0.0473 + (8) \times 0 - (-32.5) \times (-0.00046) = 0.0324 \text{ in}$$

Spring forces (interface loads):

$$R_{21} = 1588000 \times 0 = 0$$

$$R_{23} = 15300 \times 0.0324 = 496 \text{ lb (tension; = up)}$$

(Note: no spring in 2-direction.)

Point 3

Displacements:

$$u_{33} = 0.0473 + (-16.1) \times 0 - (-11.5) \times (-0.00046) = 0.0420 \text{ in}$$

Spring forces (interface loads):

$$R_{33} = 10000 \times 0.0420 = 420 \text{ lb (tension; = up)}$$

(Note: spring in 3-direction only.)

Point 4

Displacements:

$$u_{43} = 0.0473 + (16.1) \times 0 - (-11.5) \times (-0.00046) = 0.0420 \text{ in}$$

Spring forces (interface loads):

$$R_{43} = 10000 \times 0.0420 = 420 \text{ lb (tension; = up)}$$

(Note: spring in 3-direction only.)

Point 5

Displacements:

$$u_{53} = 0.0473 + (-16.1) \times 0 - (11.5) \times (-0.00046) = 0.0526 \text{ in}$$

Spring forces (interface loads):

$$R_{53} = 11200 \times 0.0526 = 589 \text{ lb (tension; = up)}$$

(Note: spring in 3-direction only.)

Point 6

Displacements:

$$u_{63} = 0.0473 + (16.1) \times 0 - (11.5) \times (-0.00046) = 0.0526 \text{ in}$$

Spring forces (interface loads):

$$R_{63} = 11200 \times 0.0526 = 589 \text{ lb (tension; = up)}$$

(Note: spring in 3-direction only.)

Point 7

Displacements:

$$u_{72} = 0 - (37.5) \times 0 = 0$$

$$u_{73} = 0.0473 + (-8) \times 0 - (37.5) \times (-0.00046) = 0.0646 \text{ in}$$

Spring forces (interface loads):

$$R_{72} = 1802000 \times 0 = 0$$

$$R_{73} = 0 \times 0.0646 = 0$$

(Note: no spring in 1-direction.)

Point 8

Displacements:

$$u_{83} = 0.0473 + (8) \times 0 - (37.5) \times (-0.00046) = 0.0646 \text{ in}$$

Spring forces (interface loads):

$$R_{83} = 0 \times 0.0646 = 0$$

(Note: spring in 3-direction only.)

Point 9

Displacements:

$$u_{93} = 0.0473 + (0) \times 0 - (57.2) \times (-0.00046) = 0.0736 \text{ in}$$

Spring forces (interface loads):

$$R_{93} = 20300 \times 0.0736 = 1494 \text{ lb (tension; = up)}$$

(Note: spring in 3-direction only.)

<u>Margin of Safety Calculation</u>

Now that the interface loads at the attachment points are known, margins of safety for the pin connections can be calculated for the given design parameters. The first step is to determine the strength of the pin connection, which will be accomplished according to the method described in the Airforce Stress Analysis Manual[3].

The material properties of 7050-T7451 (AMS 4050) aluminum alloy required for the calculation can be obtained from MMPDS[4]. The required quantities to perform the pin joint strength analysis are:

E = 10300 ksi (71016 MPa), elastic modulus
F_{tu} = 75 ksi (517 MPa), ultimate tensile strength
F_{ty} = 66 ksi (455 MPa), yield strength
F_{bru} = 111 ksi (765 MPa), ultimate bearing strength
F_{bry} = 93 ksi (641 MPa), bearing yield strength
ε_u = 9%, strain at ultimate strength

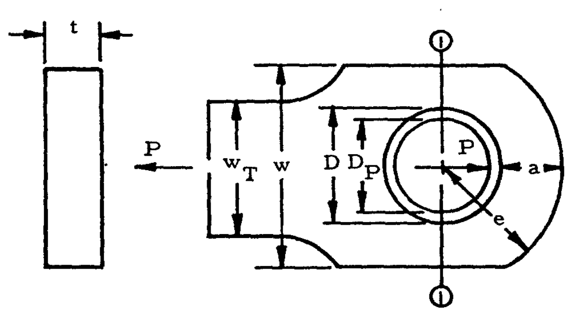
which are particularly the L-direction B-basis values for a plate stock thickness between 2 and 3 inches.

[3] AFFDL-TR-69-42, Chapter 9.
[4] MMPDS-2025, Volume 1: Conventional Materials and Joint Allowables, Batelle, 2025.

Also required is the compressive yield strength of the bushings, $F_{cy,B}$ = 60 ksi (414 MPa), and the ultimate shear and tensile strengths of the pin material, $F_{su,P}$ = 81 ksi (558 MPa) and $F_{tu,P}$ = 140 ksi (965 MPa), respectively. These values are assumed but representative of bushing and pin materials used in practice.

Note: the following calculations are based on metric properties, with the end results also given in SI units. Also, references to section and equation numbers point to the Air Force Stress Analysis Manual.

Lug



The lug geometry is as follows:

t = 0.25 in
D = 0.75 in →→→ D/t = 3
e = 0.75 in →→→ e/D = 1
w = w_T = 1.5 in →→→ D/w = 0.5
a = 0.375 in
D_P = 0.3125 in

From Figure 9-2, the value of K associated with e/D = 1 is equal to 1.6; since e/D < 1.5, the failure stress due to the bearing mode is obtained from Eq. 9-1a:

F_{bru} = K * (a/D) * F_{tu} = 1.6 * (0.375/0.75) * 75 = 60 ksi

Furthermore, since F_{tu}/F_{ty} = 1.136 < 1.304, we must use Eq. (9-3a) to compute the failure load associated with the bearing mode:

P_{bru} = F_{bru} * D * t = 60 * 0.75 * 0.25 = 11.25 kip (50 kN)

The other failure mode that must be assessed is net tension failure, which is done using the formulas of Eqs. (9-4) and (9-5) for the ultimate and yield strengths, respectively. These formulas require a coefficient K_n, given in Figure 9-4, which is based on the following parameters:

$D/w = 0.75/1.5 = 0.5$
$F_{tu}/(E * e_u) = 75/(10300 * 0.09) = 0.081$
$F_{ty}/F_{tu} = 66/75 = 0.88$

Using these numbers a value of $K_n = 0.93$ is obtained after interpolation of the graphical data. The ultimate and yield strengths of the lug then follow from Eqs. (9-4) and (9-5):

$F_{nu} = K_n * F_{tu} = 0.93 * 75 = 69.75$ ksi
$F_{ny} = K_n * F_{ty} = 0.93 * 66 = 61.38$ ksi

Since $F_{tu}/F_{ty} = 75/66 = 1.136 < 1.304$, the strength of the lug in net tension follows from Eq. (9-6a):

$P_{nu} = F_{nu} * (w - D) * t = 69.75 * (1.5 – 0.75) * 0.25 = 13.08$ kip (58.2 kN)

It can now be concluded that this particular lug fails in bearing at an applied load of 11.25 kip or 50 kN.

Clevis

The clevis geometry is as follows:

t = 0.15 in		
D = 0.5125 in	→→→	D/t = 3.417
e = 0.5125 in	→→→	e/D = 1
w = 1.025 in	→→→	D/w = 0.5
a = 0.2563 in		
D_P = 0.3125 in		

Since e/D = 1 also for the clevis, the same value of K = 1.6 is found as above for the lug. With e/D = 1 < 1.5, Eq. (9-1a) applies to produce a bearing failure strength of

$F_{bru} = 1.6 * (a/D) * F_{tu} = 1.6 * (0.2563/0.5125) * 75 = 60$ ksi

and, again from Eq. (9-3a), the bearing failure load is obtained as

$P_{bru} = F_{bru} * D * t = 60 * 0.5125 * 0.15 = 4.6$ kip (20.5 kN)

For the net section failure mode, the necessary geometrical parameters are identical to those obtained above for the lug, leading to the same values of $K_n = 0.93$ (from Figure 9-4), $F_{nu} = 69.75$ ksi, and $F_{ny} = 61.38$ ksi, and from Eq. (9-6a)

$P_{nu} = F_{nu} * (w – D) * t = 69.75 * (1.025 – 0.5125) * 0.15 = 5.36$ kip (23.8 kN)

The basic strength of the clevis is governed by the lowest value of the net section and bearing/shear-out failure load, which turns out to be 4.6 kip (20.5 kN). Note that there are actually two parts to the clevis, so that the failure load is double the value calculated above: 2 * 4.6 =9.2 kip (41 kN).

The clevis must also be checked for failure of the bushing (due to excessive bearing damage) per Section 9.3.4. According to Eq. (9-8), the ultimate bearing strength of the bushing is equal to

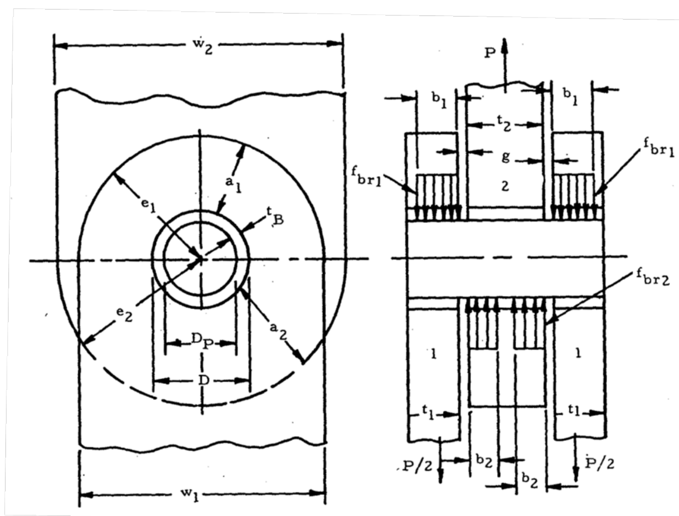
$$F_{bru,B} = 1.304 * F_{cy,B} = 1.304 * 60 = 78.24 \text{ ksi}$$

which via Eq. (9-9) leads to the bushing failure load of

$$P_{u,B} = F_{bru,B} * D_P * t = 78.24 * 0.3125 * 0.15 = 3.7 \text{ kip (16.5 kN)}$$

It turns out that the bushing governs the strength of the clevis since its failure load of 3.7 kip is less than the 4.6 kip associated with the bearing/shear-out failure.

At this point the failure loads of the individual clevis prongs and lug have been calculated as 3.7 kip and 11.25 kip, respectively. The next step is to introduce the pin and calculate the strength of the joint, as described in Section 9.4. A schematic of the joint is shown below, where the clevis prongs are labeled as member 1 and the lug as member 2.



Two distinct failure modes are typically considered for the pin: shear failure and bending failure. According to Eq. (9-12) the shear strength of the pin is obtained as follows:

$$P_{us,P} = 1.571 * D_P^2 * F_{su,P} = 1.571 * (0.3125)^2 * 81 = 12.4 \text{ kip (55.2 kN)}$$

The procedure for calculating the pin bending strength is described in Section 9.4.3. Eq. (9-15) calculates the ultimate pin bending load as

$$P_{ub,P} = 0.1963 * k_{b,P} * D_P^3 * F_{tu,P} / (t_1/2 + t_2/4 + g)$$

$$= 0.1963 * 1.56 * (0.3125)^3 * 140 / (0.15/2 + 0.25/4 + 0.2) = 3.9 \text{ kip}$$

where the suggested value of $k_{b,P}$ = 1.56 for reasonably ductile materials has been used. Then, the failure load of the joint follows from Eq. (9-16) as

$$C = P_{u,L,b,1} * P_{u,L,b,2} / (P_{u,L,b,1} * t_2 + P_{u,L,b,2} * t_1)$$

$= 3.7 * 11.25 / (3.7 * 0.15 + 11.25 * 0.25) = 12.36$ kip/in

$P_{ub,P\ max} = 2 * C * \sqrt{[\ P_{ub,P}/C * (t_1/2 + t_2/4 + g) + g^2\ ]} - 2 * C * g$

$= 2 * 12.36 * \sqrt{[\ 3.9/12.36 * (0.15/2 + 0.25/4 + 0.2) + (0.2)^2\ ]} - 2 * 12.36 * 0.2$

$= 4.5$ kip (20 kN)

where

$P_{u,L,b,1} = 3.7$ kip (16.5 KN), and

$P_{u,L,b,2} = 11.25$ kip (50 kN)

are the previously calculated failure loads for the clevis and lug members, respectively.

Now that the strength of the pin joint is known, the associated reserve factor, RF, (or margin of safety, MS) can be calculated by comparison to the applied load to the joint. The applied load at interface point 9 was calculated as $R_{93} = 1496$ lb (6.65 kN). Since the applied load resultants (those following from the aerodynamic pressure distribution) are specified as *limit* loads, a safety factor of 1.5 must be applied to obtain *ultimate* loads according to 14 CFR 25.303 (EASA CS 25.303). In addition, a fitting factor of 1.15 is applied per 14 CFR 25.625 (EASA CS 25.625).

The calculated joint strength is 4.2 kip (18.7 kN)

Hence, the resulting load that the pin joint at point 9 must be designed for is

$1494 * 1.5 * 1.15 = 2577$ lb (11.5 kN)

Therefore

RF = 4.2/2.6 = 1.62 ; MS = RF – 1 = 1.62 – 1 = 0.62